Fuzzy Modeling of Nonlinear Stochastic Systems by Learning from Examples

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Problem Statement:

Fuzzy Learning From Examples

- Input output data pairs

\[
\begin{bmatrix}
  x_1^{(p)} \\
  x_2 \\
  \vdots \\
  x_m
\end{bmatrix}
\begin{bmatrix}
  y_1^{(p)} \\
  y_2 \\
  \vdots \\
  y_n
\end{bmatrix}
\]

\( p=1,2,\ldots,N \)

Unknown System

Fuzzy System
Motivation

- Fuzzy Systems, due to their universal approximation property, are a good framework for modeling complex and highly nonlinear systems. However, existing fuzzy modeling methods assume a deterministic method and so interpret inconsistencies in the training data as unwanted noise. This is while in fuzzy modeling of the systems with random nature, the above inconsistency is part of the system and should be taken into account.
A new modification to one of the standard fuzzy modeling methods (Table look up scheme) is presented here. The method takes the randomness into account by considering the statistical properties of the training dataset.
PART I

Table Look-up Scheme
5 Step learning algorithm by Wang, generates fuzzy rules with fixed membership functions

Consider a two input one output system for example

\[(x_1^p, x_2^p, y^p) \quad p=1,2,\ldots,N\]
STEP 1:

Fuzzy Partitioning of the input and output variables

MF1_1, MF1_2, MF1_3, MF1_4, MF1_5

MF2_1, MF2_2, MF2_3, MF2_4, MF2_5

OMF1, OMF2, OMF3, OMF4, OMF5
STEP 2:

Generate fuzzy rules for each of the given data pairs: (resulting the initial fuzzy rule base)

\[(x_1^p, x_2^p, y^p) \implies \text{Rule } p^{\text{p}}\]

\[p=1,2,...,N\]

\[
\begin{align*}
\mu_1(x_1^p) & \quad \mu_1(x_2^p) & \quad \mu_1(y^p) & \quad \text{If } x_1 \text{ is MF}_1^2 \text{ and } x_2 \text{ is MF}_1^4 \\
\mu_2(x_1^p) & \quad \mu_2(x_2^p) & \quad \mu_2(y^p) & \quad \text{then } y \text{ is OMF}_1^1 \\
\mu_3(x_1^p) & \quad \mu_3(x_2^p) & \quad \mu_3(y^p) \\
\mu_4(x_1^p) & \quad \mu_4(x_2^p) & \quad \mu_4(y^p) \\
\mu_5(x_1^p) & \quad \mu_5(x_2^p) & \quad \mu_5(y^p)
\end{align*}
\]
How to assign Membership functions to data pairs?

The fuzzy variable with maximum membership value is associated with $x_2^p$. 
STEP 3:

Assign a degree to each of the resulting rules:

\[(x_1^p, x_2^p, y^p) \implies \text{Rule } p, \text{ Degree(Rule } p)\]

\[p=1,2,...,N\]

If \(x_1\) is A and \(x_2\) is B then \(y\) is C

\[\text{Degree} = \mu_A(x_1) \times \mu_B(x_2) \times \mu_C(y)\]
• $N$ is usually much greater than $R_{\max} = 5 \times 5 = 25$

If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^1$ degree1

If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^1$ degree2

If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^1$ degree3

If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^2$ degree4

If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^2$ degree5

If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^3$ degree6

Redundant

Inconsistent
Creating final fuzzy rule base by removing inconsistent and redundant rules

\[ \text{Rule } p \quad \Rightarrow \quad \text{Rule } q \]

\[ p=1,2,...,N \quad q=1,2,...,K \quad K \leq R_{\text{maz}} \]
Removing Inconsistencies in *Standard Method*

<table>
<thead>
<tr>
<th>If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^1$</th>
<th>degree1</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^1$</td>
<td>degree2</td>
</tr>
<tr>
<td>If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^1$</td>
<td>degree3</td>
</tr>
<tr>
<td>If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^2$</td>
<td>degree4</td>
</tr>
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<td>degree5</td>
</tr>
<tr>
<td>If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^3$</td>
<td>degree6</td>
</tr>
</tbody>
</table>

Maximum degree

$\text{degree 6} = \max \{\text{degree 1, degree 2, degree 3, degree 4, degree 5, degree 6}\}$
Reliability Factor

For each given set of $k$ rules with the same antecedent parts, reliability factor is defined as:

\[
\text{Reliability Factor} = \frac{k_1}{k}
\]

Where:

- $k_1$ is the number of redundant rules
- $k$ is total number of the redundant and inconsistent rules (having the same antecedent part)

The reliability factor is used as a weighting factor for rule degree in modified method.
Removing Inconsistencies in \textit{Modified Method}

- If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^1$ \(\deg_1 \times \frac{3}{6}\)
- If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^1$ \(\deg_2 \times \frac{3}{6}\)
- If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^1$ \(\deg_3 \times \frac{3}{6}\)
- If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^2$ \(\deg_4 \times \frac{2}{6}\)
- If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^2$ \(\deg_5 \times \frac{2}{6}\)
- If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^3$ \(\deg_6 \times \frac{1}{6}\)

If $x_1$ is $A$ and $x_2$ is $B$ then $y$ is $C^2$ \((\deg_4 + \deg_5)/2\)
STEP 5:

Determining the overall fuzzy system through selecting a defuzzification method

- Membership functions: Are defined in Step 1
- Fuzzy Rule Base: is defined in Step 4
- Any T-norm and S-norm can be defined
- A defuzzification method such as Centroid is selected

=> Fuzzy System is defined
Summary

• Modified Table-look-up Scheme treats differently with *inconsistency* and *redundancy* in initial fuzzy rule set.

• Indeed it takes the statistical properties of the observed system into account through introducing a new concept of *Reliability Factor*
PART II

Mackey-Glass Time Series Prediction
Mackey-Glass Time Series

- Mackey-Glass is a chaotic time series which is used as a benchmark for prediction and modeling problems.

Defining Equation:

\[
\frac{dx(t)}{dt} = \frac{0.2x(t - \tau)}{1 + x^{10}(t - \tau)} - 0.1x(t)
\]
A Sample Stochastic-Chaotic Mackey Glass Series

\[
\frac{dx(t)}{dt} = \frac{0.2x(t - \tau)}{1 + x^{10}(t - \tau)} - 0.1x(t) + n(t)
\]

\[T_s = \text{sampling time}\]
\[\text{var}(n) = \text{Noise power}\]
Prediction of the time series is possible through fuzzy modeling of a system with past samples as inputs and current or future samples as output.
Validating the model

Scaling

Fuzzy Model of the system

\[ x(t) \quad 700 < t < 1000 \]

\[ x_s(t-1) \]

\[ x_s(t-4) \]

\[ \hat{x}_s(t) \]

\[ e(t) \]

\[ SAE = \sum_t |error(t)| \]

\[ SASE = \sum_t |error(t)|^2 \]

\[ \sum_{t=1}^n |error(t)| \]

normalized SAE \% = \frac{\sum_{t=1}^n |error(t)|}{n} \times 100
PART III
Simulation Results
Simulation Results

Number of Membership Functions for Each Variable

- Standard Mackey Glass Series: Standard Table Look-up Scheme - 7
- Randomized Mackey Glass Series:
  - Standard Table Look-up Scheme: 3
  - Modified Table Look-up Scheme:
    - 7
4-inputs, 7MFs, 700 Training data

Results:

45 Rules
Error=4.6%
4-inputs, 7MFs, 700 Training data

**Standard Method:**

- 117 Rules
- Error=7.9%

**Modified Method:**

- 117 Rules
- Error=6.8%
4-inputs, 3MFs, 700 Training data

**Standard Method:**
31 Rules
Error=14.2%

**Modified Method:**
31 Rules
Error=8.4%
## Results:

\[ \text{Cov}(n(t)) = 2 \]

<table>
<thead>
<tr>
<th></th>
<th>7 MF’s</th>
<th>3 MF’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>SASE</td>
<td>7.9%</td>
<td>14.2%</td>
</tr>
<tr>
<td>(Standard Algorithm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SASE</td>
<td>6.8%</td>
<td>8.4%</td>
</tr>
<tr>
<td>(Modified Algorithm)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Rule numbers:

- 117
- 31
The effect of noise power

\[
\frac{dx(t)}{dt} = \frac{0.2x(t-\tau)}{1 + x^{10}(t-\tau)} - 0.1x(t) + n(t)
\]

Increasing the noise power
The effect of noise power, 3MFs

Comparison of %error for standard and modified algorithms (3 MF inputs)

Consistently is better than standard method
Comparison of % error for standard and modified algorithms (7 MF inputs)

Consistently is better than standard method.
• Modified Table Look-Up Scheme (as a deterministic modeling algorithm for randomized systems) is superior than standard method especially when:

1- The number of allowed rules is smaller.

2- The randomness nature of the system is more significant.

• This superiority is more consistent with noise power variation when the number of allowed rules are greater.
THE END